Simple bets to elicit private signals

Yan Xu

Erasmus University Rotterdam and Tinbergen Institute

Aurelien Baillon

Erasmus University Rotterdam and Tinbergen Institute

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Task: elicit private information from crowds

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Why?

- gather knowledge and guide decision-making
  - opinions and attitudes from social-economic surveys
  - tastes and experiences on reviews sites like Rotten Tomatoes/Yelp/TripAdvisor ...
  - evaluations and recognitions on crowdsourcing platforms

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Goal:

- elicit **high-quality** information from crowds
  - participation
  - informativeness (more efforts)
  - truth-telling

Task: elicit high-quality information from crowds

How?

design payment schemes to **align incentives** with participation, more efforts and truth-telling.

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Two categories of elicitation methods:

- single DM setting with the verifiable truth: condition payment with an exogenous event/item
  - proper scoring rule: realization of the underlying event
  - BDM: allocation of the underlying item
- game setting with the unverifiable truth: condition payment with an surrogate event/item
  - peer prediction: peers' reports
  - Bayesian truth serum: peers' reports and predictions
  - Bayesian market: transaction of an "opinion" asset

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Challenge:

- practice: complicated and non-transparent
- theory: common prior, homogeneous and risk neutral agents

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This paper:

- two mechanisms: top-flop and target betting
- In the single DM setting with verifiable truth, truth-telling is incentive compatible.
- In the game setting with unverifiable truth, truth-telling can be an equilibrium w/o common prior, homogeneity and risk neutral assumptions.

examples of top-flop betting and target betting

An example of top-flop betting

- attend the premiere of a new "Avengers X" movie
- choose one from two bets
    1. (top bet): "Avengers X" will have a **higher** rotten tomato score than another random superhero movie.
    2. (flop bet): "Avengers X" will have a **lower** rotten tomato score than another random superhero movie.
- get a prize if your bet wins

Intuitive results for top-flop betting

Will you participate in this bet?

Which bet will you choose?

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- if you **like** “Avengers X”
- if you do **not like** “Avengers X”

With top-flop betting, respondents will participate and reveal private signals through betting choices.

- positive signal $\Rightarrow$ bet “top”
- negative signal $\Rightarrow$ bet “flop”

An example of target betting

- after watching a new “Avengers X” movie
- choose one from two bets to win a prize
  1. bet the new “Avengers X” movie will have a rotten tomato score **higher** than 0.8.
  2. bet another random superhero movie will have a rotten tomato score **higher** than 0.8.

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- positive signal ⇒ bet the new "Avengers X" movie
- negative signal ⇒ bet another random superhero movie

When does top-flop and target betting (not) work?

Set-ups

- a collection of items $\mathcal{K} = \{1, 2, ..., K\}$
- each $k \in \mathcal{K}$ has a score Y_k (e.g. RT score, box office)
- $\forall k \in \mathcal{K}$, the bettor has a prior $H_k(\cdot)$
- for item i, the bettor has a private signal $t_i \in \{1, 0\}$
- the center randomly chooses an item $j \neq i$ for the bettor and formulates top flop bets or target-y bets
  - top bet: $y_i > y_j$
  - flop bet: $y_i < y_j$
  - bet on item i: $y_i > y$
  - bet on item j: $y_j > y$
- the prize for a winning bet is $\pi \succ 0$

Assumptions on signal technologies

Assumption 1

$Pr(t_i = 1|Y_i > \delta) > Pr(t_i = 1|Y_j > \delta)$ for all $j \in \mathcal{K} \setminus \{i\}$.

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- interpretation: signals are assorted

	signal	**news**
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- counter-example: irrelevant Y_i and Y_j
  - $Y_i > \delta$: movie i's director is taller than 175cm
  - $Y_j > \delta$: movie j's director is taller than 175cm

  bet on/against movie i's director is taller than movie j's?

- in practice: formulate bets on signal-assorted scores

A special case on signal technologies

When Y_j is uninformative: $Pr(t_i = 1|Y_j) = Pr(t_i = 1)$

Assumption 1'

$Pr(t_i = 1|Y_i = y_i)$ is strictly increasing in y_i

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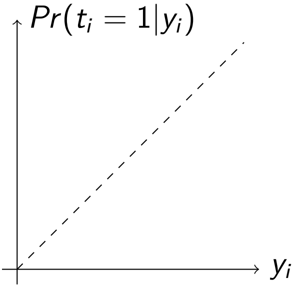

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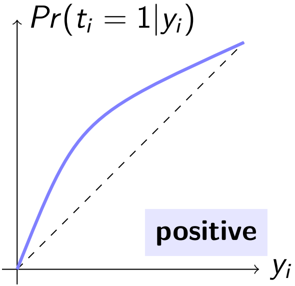

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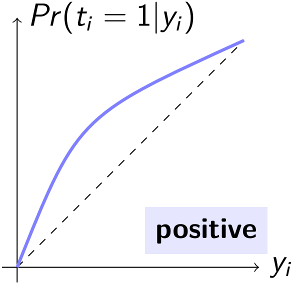

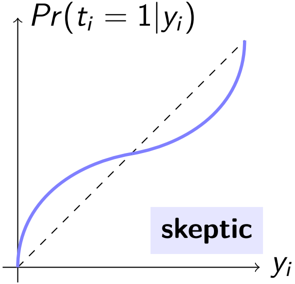

Assumption on priors

Assumption 2

Bettor has the same non-degenerate prior $H(\cdot)$ about Y_k for all $k \in \mathcal{K}$

- interpretation: items in $\mathcal{K}$ are "informationally non-differentiable" ex-ante
- counter-example: item i and j from different collections bet on/against a blockbuster vs. a random independent movie
- in practice: select similar items for the collection

Assumptions on independence and preference

Assumption 3

For any $i, j \in \mathcal{K}$, Y_i, Y_j are independent and conditionally independent given signals.

- necessary for top-flop betting, but not target betting
- relaxation requires specifications of priors and signal technologies.

Assumption 4

Bettors are probabilistically sophisticated.

- Bettors choose the bet that has the highest chance to get π
- Bettors can be risk averse, non-expected utility,...

A single bettor: betting on exogenous scores

Truth-telling in a top-flop betting

Theorem 1

Under assumption 1 to 4, a bettor will participate in the top-flop betting for any $j \in \mathcal{K} \setminus \{i\}$ and will be truthful.

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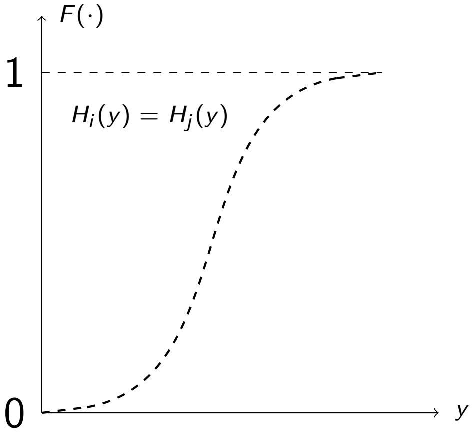

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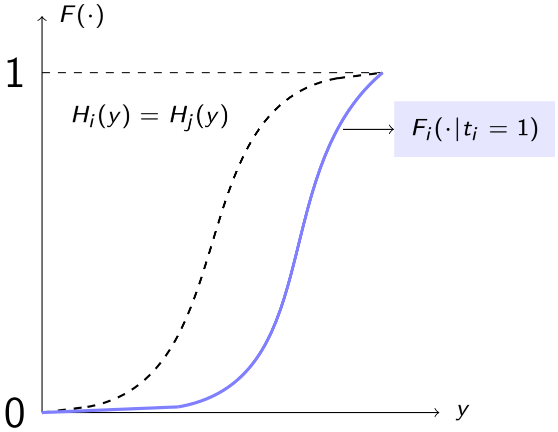

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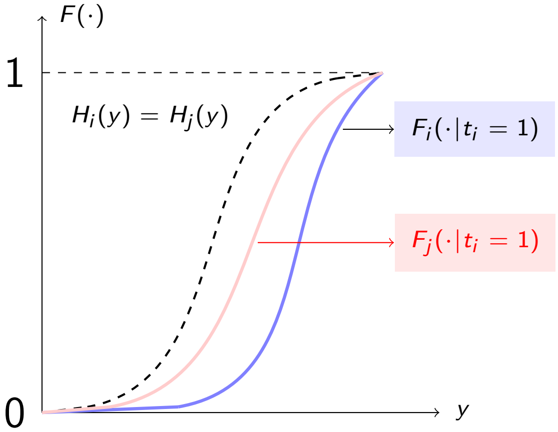

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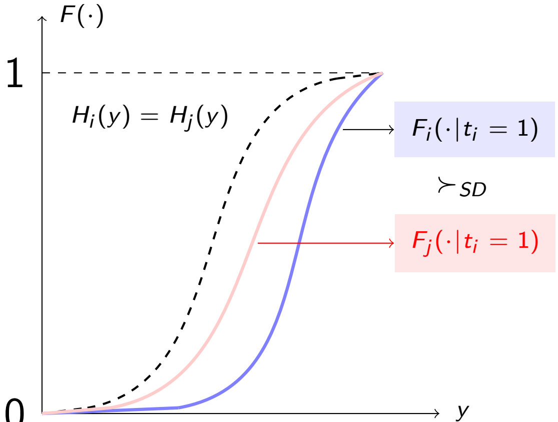

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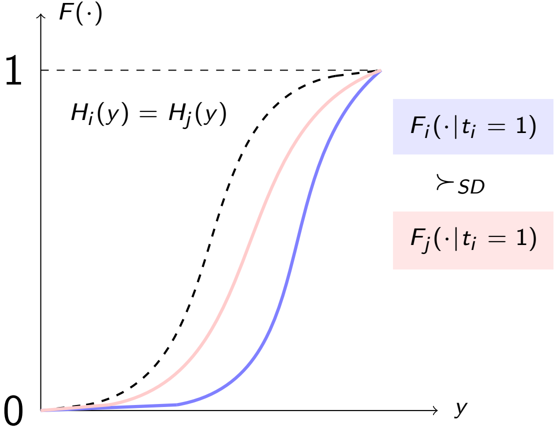

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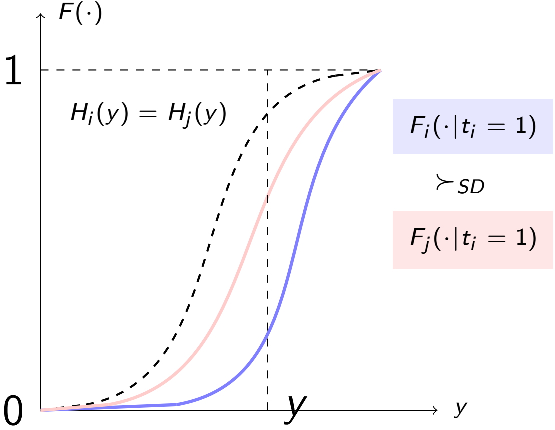

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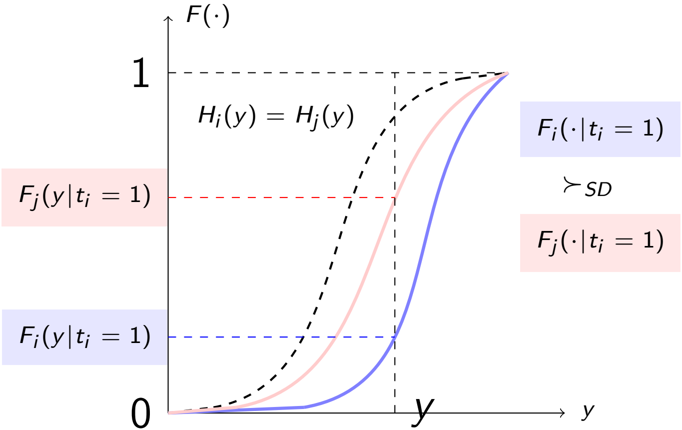

Game setting: betting on endogenous scores

Top-flop in a game setting

- agents receive signals and simultaneously choose between
  - top bet: $y_i > y_j$
  - flop bet: $y_i < y_j$
- y_i is the proportion of top choices when item i is involved

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It is common knowledge that Assumption 1-4 hold for all agents.

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It is common knowledge that Assumption 1-4 hold for all agents.

- common interpretation:
- Agents may think everyone is different in priors, signal technologies, and thus posteriors.
- Agents agree that the FOSD between posteriors and priors holds for all.

Truth-telling equilibrium for a top-flop game?

Without common prior, BNE concept does not apply.

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Alternative equilibrium concepts: Nash Equilibrium

- transform the betting game with incomplete information to a strategic betting game with imperfect information
- redefined preferences are over lotteries, and are type-dependent and individual-specific
- verify that truth-telling is a strict pure strategy Nash equilibrium

A top-flop betting game with 4 agents

- 4 agents participate the game: $\mathcal{N} \equiv \{A, B, C, D\}$
- A, B receive signals from item i and C, D receive signals from item j: $(t_A^i, t_B^i, t_C^j, t_D^j) \in \Pi_{n\in\mathcal{N}}\{1, 0\}$
- For any agent $n \in \mathcal{N}$, his strategy profile is $a_n = (a_n^0, a_n^1)$.
- There are 4 pure strategy profiles: truth-telling (0,1); lying (1,0); always top (1,1) and always flop (0,0).
- Payoffs for two bets are:
  - top bet: receive π if $(a_B, a_C) = (1, 0)$, nothing otherwise
  - flop bet: receive π if $(a_B, a_C) = (0, 1)$, nothing otherwise
- Check that truth-telling $a = (a^0, a^1) = (0, 1)$ is a strict pure strategy Nash equilibrium.

Truth-telling as a Nash equilibrium

Suppose A's signal is 1, payments for reporting 1 and 0 are:

signal (t_B^i, t_C^j)	**action** (a_B, a_C)	**Top** $a_A^1 = 1$	**Flop** $a_A^1 = 0$	**posterior** $P_A(t_B^i, t_C^j \mid t_A^i)$
(1,0)	(1,0)	π	0	$P_A(1, 0 \mid 1)$
(0,1)	(0,1)	0	π	$P_A(0, 1 \mid 1)$
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(1,1)	(1,1)	0	0	$P_A(1, 1 \mid 1)$

- Under assumption 1-3 with $y_i = t_B^i$ and $y_j = t_C^j$, $P_A(t_B^i = 1, t_C^j = 0 | t_A^i = 1) > P_A(t_B^i = 0, t_C^j = 1 | t_A^i = 1)$
- If agent A's type is 0, he will bet flop.
- Truth-telling is a strict pure strategy Nash equilibrium.

Conclusion

Top-flop and target betting can elicit informative and unverifiable truth

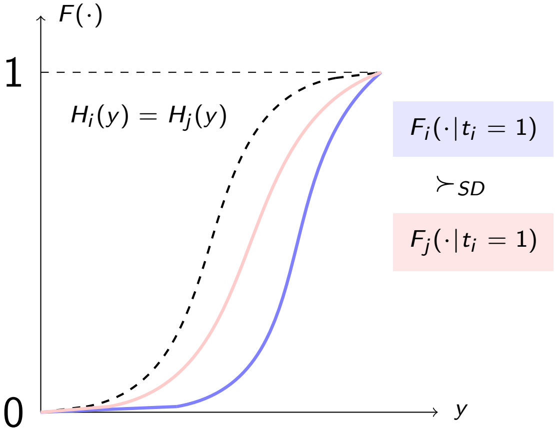
- simple and transparent
- relax heavy assumptions

Thank you for your attention!

Appendix

Implication for incentivizing efforts

- what if it is costly to obtain private signals?
- more efforts ⇒ more precise signals



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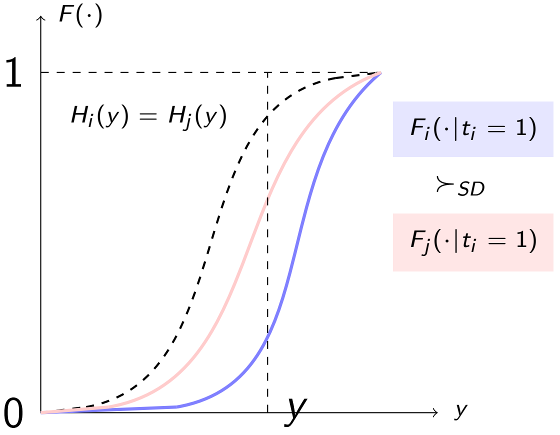

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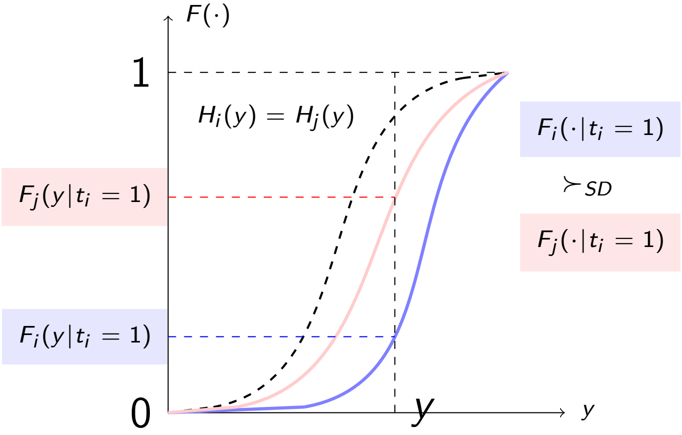

References

- single DM setting with verifiable truth
  - proper scoring rule: Winkler 69; Schotter & Trevino 14.
  - BDM: Becker et al. 64; Karni 09.

- game setting with unverifiable truth
  - peer prediction: Miller et al. 05; Jurca & Faltings 06, 09; Gao et al. 14;...
  - Bayesian truth serum: Prelec 04; Weaver & Prelec 13; Witkowski & Parkes 12; Radanovic & Faltings 13, 14,...
  - Bayesian market: Baillon 17.